Improvement of a Method to Compute the Inductance Matrix of Multilayer Transformer Windings for Very Fast Transients

P. Gómez, *Member, IEEE*, F. de León, *Senior Member, IEEE*, and F. P. Espino-Cortés, *Member, IEEE*

***Abstract*—This letter presents a simple and practical enhancement of a previously published method, based on the method of images, for the computation of the inductance matrix of transformer windings for high-frequency transients. It is first shown that this method can present problems when applied to mutilayer windings. Then, a modification that solves the problem is proposed and validated.**

***Index Terms*—Inductance matrix, magnetic fields, method of images, transformer windings, very fast transients.**

I. Introduction

A RECENT paper has proposed a method for the computation of the inductance matrix for modeling transformer windings for very fast electromagnetic transients [1]. The matrix is obtained by means of an algorithm based on the method of images. Results on a test case showed that when compared with finite-element method (FEM) simulations, the proposed method gives very accurate results. However, it is shown in this letter that when applied to a winding consisting of multiple layers of windings, the method can present inconsistencies.

A simple modification is proposed here to improve the performance of the method for multilayer windings. A test case shows that this proposal results in substantial accuracy improvements.

II. Problem Statement

It was observed that the application of the method described in [1] to a transformer with several winding layers results in a nonsymmetrical inductance matrix. The computed values do not comply with $L_{ij} = L_{ji}$ in some cases. FEM simulations for the same transformer, when the inductance is computed by integrating the flux linkage (the basis of the method described in [1]), present the same issue. In simple terms, when L_{ij} is obtained by exciting conductor i and computing the flux linked by conductor j, the result is different than when exciting conductor j and computing the flux linked by conductor i (L_{ji}).

Manuscript received December 11, 2012; accepted February 13, 2013. Date of publication March 14, 2013; date of current version March 21, 2013. This work was supported by Secretaría de Investigación y Posgrado, Instituto Politécnico Nacional (SIP-IPN) and Consejo Nacional de Ciencia y Tecnología (CONACyT, Project: 154969). Paper no. PESL-00173-2012.

P. Gómez and F. P. Espino-Cortés are with the Electrical Engineering Department, SEPI-ESIME Zacatenco, Instituto Politécnico Nacional (IPN), Mexico City C. P. 07738, Mexico (e-mail: pgomezz@ipn.mx).

F. de León is with the Department of Electrical and Computer Engineering, Polytechnic Institute of New York University, Brooklyn, NY 11201 USA (e-mail: fdeleon@poly.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TPWRD.2013.2248961

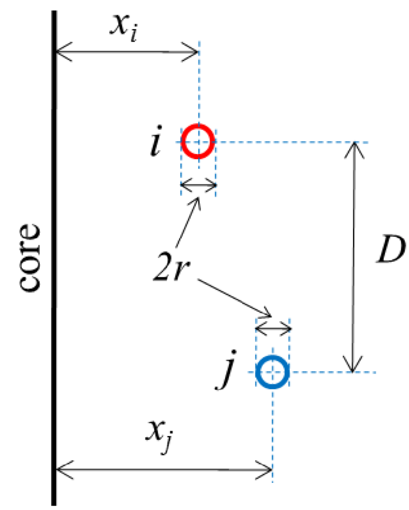

Fig. 1. Geometrical disposition of conductors.

To explore the reason for this discrepancy, the region of the winding referred to in [1] as *outside the core window* is considered. In this region, L_{ij} is computed from the method of images as

$$L_{ij} = \frac{\mu_0}{4\pi} \ln \frac{(x_j + x_i - r)^2 + D^2}{(x_j - x_i + r)^2 + D^2} \tag{1}$$

which corresponds to [1, (5)]. (Please note that this equation should read 4π instead of 2π in the denominator.) In (1), x_i and x_j are the horizontal distances from the core wall to the center of conductors i and j, respectively. D is the vertical distance between the conductors and r is their radii, as illustrated in Fig. 1. On the other hand, L_{ji} is obtained as

$$L_{ji} = \frac{\mu_0}{4\pi} \ln \frac{(x_j + x_i - r)^2 + D^2}{(-x_j + x_i + r)^2 + D^2}. \tag{2}$$

It can be noticed that L_{ij} from (1) is not equivalent to L_{ji} from (2) since, in their denominators

$$(x_j - x_i + r)^2 \neq (-x_j + x_i + r)^2. \tag{3}$$

Both sides of (3) would be equal only if $x_i = x_j$ or $r = 0$. The former is true only if the conductors correspond to turns from the same winding layer. The latter is, of course, never true; however, when the turns are sufficiently apart from each other, r can be safely neglected.

To summarize, computing the mutual inductance using the method of images based on flux linkages can give erroneous results when the conductors are not aligned with respect to the reference plane (in this case, the core leg). Errors can also be expected when the cross-sectional area of the conductors cannot be neglected (i.e., the conductors are in very close proximity). Therefore, the problem of obtaining a nonsymmetric inductance matrix from the method of images may appear for multilayer and disc-type windings.

III. Solution

When the magnetic energy method is applied in FEM, a symmetric inductance matrix is obtained by definition. On the other hand, in the region referred to in [1] as *inside the core window*, a similar problem arises. This is because the formulae obtained for this region, consisting of the conductor and eight images to take the four core walls and the four corners into account [1, (10a)–(10i)]), are obtained in a similar manner than those *outside the core window*. However, by analyzing these equations, it can be demonstrated that making $r = 0$ does not necessarily result in $L_{ij} = L_{ji}$. Therefore, the modification proposed in this letter for the mutual inductance between unaligned conductors (from different layers of the winding) inside the core window is divided into two parts:

1) *Neglecting the cross-sectional area of the conductors* (setting $r = 0$), considering that this can give a good approximation (in terms of the magnetic flux distribution) if the distance between conductors is significantly larger than their radius, which is the case for conductors from different winding layers.
2) *Considering the conductor closer to the left side of the core (inner conductor) as the excited conductor for the method of images*. This results in a longer integration contour for the computation of linked flux than the one obtained when exciting the outer conductor. As a consequence, the excess flux integrated by setting $r = 0$ is smaller than that obtained by integrating along the shorter contour.

IV. Test Case

The same geometry defined in [1, Fig. 8] is considered. However, for the purpose of validating the proposal of this letter, two new layers are included, each equal to the first layer (same type and number of conductors as well as the distance between them). The distance between layers is 1 cm. Turns from layers 1, 2, and 3 are numbered 1 to 30, 31 to 60, and 61 to 90, respectively (top to bottom). Mutual inductances between conductor 1 from layer 1 and different conductors from layers 2 and 3 are computed, as shown in Table I. Values from this table are defined as follows:

1) *Energy method*. Computed from FEM simulations and used as the base solution for the validation of the method of images.
2) *Method of images 1*. Computed using the method described in [1] exciting conductor 1.
3) *Method of images 2*. Computed using the method described in [1] exciting conductor j, which takes different values from 31 to 90.
4) *Method of images 3*. Computed using the method described in this paper: considering a negligible conductor radius $(r = 0)$ and always exciting the inner conductor.

TABLE I
Percent Difference in Mutual Inductance

Mutual inductance		Energy method	Method of images 1	Method of images 2	Method of images 3
		[μH]	[%]		
Between layers 1 & 2	$L_{1,31}$	0.2507954	-14.86	20.74	3.26
	$L_{1,32}$	0.2170363	-13.42	10.84	1.29
	$L_{1,33}$	0.1559936	-11.45	3.57	0.20
	$L_{1,34}$	0.1093151	-10.73	0.10	-0.20
	$L_{1,35}$	0.0772709	-10.61	-1.87	-0.35
	$L_{1,40}$	0.0165853	-11.10	-5.23	0.21
	$L_{1,50}$	0.0012049	-12.10	-6.84	-0.82
	$L_{1,60}$	0.0000875	-11.79	-6.54	-0.43
Between layers 1 & 3	$L_{1,61}$	0.1490428	-14.86	12.36	0.57
	$L_{1,62}$	0.1458883	-15.09	8.96	-0.70
	$L_{1,63}$	0.1239072	-14.39	5.16	-1.41
	$L_{1,64}$	0.0979875	-13.68	2.31	-1.72
	$L_{1,65}$	0.0752463	-13.22	0.34	-1.85
	$L_{1,70}$	0.0192202	-12.85	-3.60	-1.85
	$L_{1,80}$	0.0014612	-12.88	-4.69	-1.68
	$L_{1,90}$	0.0001055	-11.68	-3.44	-0.31
Average of absolute value			**12.80**	**6.04**	**1.05**

Several interesting conclusions arose from Table I as follows.

- Integrating the shorter contour (method of images 1) results in significant underestimation of the mutual inductances for all of the values computed.
- Integrating the longer contour (method of images 2) gives better results in most cases and in average. However, results are considerably larger than those from FEM when the conductors are in close proximity. These would actually be the most important mutual inductance values for transient analysis, since they correspond to the higher magnetic coupling between turns.
- Integrating the longer contour and setting $r = 0$ gives, by far, the best results in most cases and in average. Additional computations were performed for the closer and farther layers (not shown in this letter). The results were consistent with the proposed approach, giving the closest results to the energy method.

V. Conclusion

This letter has proposed an improvement of an existing method for the computation of the inductance matrix of transformer windings for high-frequency transients. The required modifications are easy to include in the original method, and the accuracy is largely improved for multilayer windings. Since the method is intended for its future application in the implementation of transformer winding models in EMTP-type programs, it is essential that it be kept as simple and practical as possible.

References

[1] P. Gómez and F. de León, “Accurate and efficient computation of the inductance matrix of transformer windings for the simulation of very fast transients,” *IEEE Trans. Power Del.*, vol. 26, no. 3, pp. 1423–1431, Jul. 2011.